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Description of the data generating system utilized in “Prediction-Error Identification of LPV Systems: A Nonparametric Gaussian Regression Approach”

By

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I. INTRODUCTION

In this report, we give the exact coefficient function matrices \(a_i, b_j\) of the Linear Parameter-Varying (LPV) process model utilized in [1], together with the coefficient function matrices \(c_i, d_j\) of the considered noise dynamics, i.e., the corresponding full Box Jenkins (BJ) model.

II. LPV-BJ MODEL DESCRIPTION

Consider a multi-input multi-output (MIMO) data generating LPV system described in discrete-time by the following difference equations:

\[
A_0(p, k, q^{-1})\hat{y}(k) = B_0(p, k, q^{-1})u(k),
\]

\[
D_0(p, k, q^{-1})v(k) = C_0(p, k, q^{-1})e(k),
\]

\[
y(k) = \hat{y}(k) + v(k),
\]

where \(k \in \mathbb{Z}\) is the discrete time, \(q\) is the forward time-shift operator, i.e., \(qx(k) = x(k+1)\), \(u : \mathbb{Z} \to \mathbb{U} = \mathbb{R}^{n_u}\) is the input, \(\hat{y}, y : \mathbb{Z} \to \mathbb{Y} = \mathbb{R}^{n_y}\) are the noiseless and noisy outputs respectively, \(p : \mathbb{Z} \to \mathbb{P}\) is the so-called scheduling variable with compact range \(\mathbb{P} \subseteq \mathbb{R}^{n_p}\), \(v : \mathbb{Z} \to \mathbb{Y}\) is a coloured noise process, and \(e : \mathbb{Z} \to \mathbb{Y}\) is a white noise process with normal (Gaussian) distribution, i.e., \(e(k) \sim \mathcal{N}(0, \Sigma_e)\) with covariance \(\Sigma_e \in \mathbb{R}^{n_y \times n_y}\). The \(p\)-dependent operators \(A_0(p, k, q^{-1})\) and \(B_0(p, k, q^{-1})\) that describe the process model (1a) are matrix polynomials in \(q^{-1}\) of degree \(n_a\) and \(n_b\) respectively:

\[
A_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_a} a_i(p, k, i)q^{-i},
\]

\[
B_0(p, k, q^{-1}) = \sum_{j=0}^{n_b} b_j(p, k, j)q^{-j},
\]

where \(I_{n_y}\) is the \(n_y\)-dimensional identity matrix and the matrix functions \(a_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_y}\) and \(b_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_u}\) are shorthand notations for \(a_i(p, k, i) = a_i(p(k), \ldots, p(k-i))\) and \(b_j(p, k, j) = b_j(p(k), \ldots, p(k-j))\). These functions are assumed to be smooth and bounded functions on \(\mathbb{P}\). In a similar fashion, for the noise model (1b), the relations \(D_0(p, k, q^{-1})\) and \(C_0(p, k, q^{-1})\) are defined as

\[
C_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_c} c_i(p, k, i)q^{-i},
\]

\[
D_0(p, k, q^{-1}) = I_{n_y} + \sum_{j=1}^{n_d} d_j(p, k, j)q^{-j},
\]

where \(d_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_y}\) and \(c_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_y}\) are the coefficient functions matrices of the monic polynomials matrices in \(q^{-1}\) of degree \(n_c\) and \(n_d\), respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with \(n_y = 2, n_u = 2, n_p = 2\). The full description of this model is given below.
III. COEFFICIENT FUNCTIONS OF THE PROCESS DYNAMICS

\[ b_0(p, k, 0) = \begin{bmatrix} 1 - \exp(-0.6p_1(k)) & 0.64 - 0.72 \exp(0.7p_1(k)) \\ 0.3 - 0.4p_1^2(k) + 0.5p_2(k) & 0.2 + 0.98 \tan^{-1}(0.66p_2(k)) \end{bmatrix} \quad (4a) \]

\[ b_1(p, k, 1) = \begin{bmatrix} 0.24 - 0.32p_1^2(k) + 0.4p_2(k - 1) & 0.22 \exp(0.4p_1(k - 1)) \\ 0.16 + 0.9 \tan^{-1}(0.63p_2(k)) & 0.22 - 0.5p_1^2(k) + 0.45p_2(k - 1) \end{bmatrix} \quad (4b) \]

\[ b_2(p, k, 2) = \begin{bmatrix} 0.16 + 0.64 \tan^{-1}(0.8p_2(k - 2)) & 0.14 + 0.7 \tan^{-1}(0.6p_2(k - 2)) \\ 0.64 - 0.64 \exp(-0.6p_1(k - 1)) & 0.17 - 0.32p_1^2(k) + 0.32p_2(k - 1) \end{bmatrix} \quad (4c) \]

\[ a_1(p, k, 1) = \begin{bmatrix} 0.2 + 0.12p_2^2(k - 1) & 0 \\ 0 & 0.2 + 0.35 \tan^{-1}(p_1(k)) \cos(p_1(k - 1)) \end{bmatrix} \quad (4d) \]

\[ a_2(p, k, 2) = \begin{bmatrix} 0.19 + 0.15 \tan^{-1}(p_1(k - 1)) \cos(p_2(k - 2)) & 0 \\ 0 & 0.17 + 0.11p_2^2(k - 1) \end{bmatrix}. \quad (4e) \]

IV. COEFFICIENT FUNCTIONS OF THE NOISE DYNAMICS

\[ d_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.3 \sqrt{|p_1(k)|} & 0 \\ 0 & 0.45 + 0.45 \sin(p_2(k)) \end{bmatrix} \quad (5a) \]

\[ d_2(p, k, 2) = \begin{bmatrix} 0.34 + 0.34 \sin(p_2(k - 1)) & 0 \\ 0 & 0.23 + 0.23 \sqrt{|p_1(k - 2)|} \end{bmatrix} \quad (5b) \]

\[ c_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.45p_1^2(k) + 0.3p_2^2(k - 1) & 0 \\ 0 & 0.3 + 0.45p_2^2(k - 1) \end{bmatrix} \quad (5c) \]

\[ c_2(p, k, 2) = \begin{bmatrix} 0.24 + 0.36p_1^2(k - 1) & 0 \\ 0 & 0.24 + 0.36p_2^2(k - 2) + 0.24p_2^2(k - 1) \end{bmatrix}. \quad (5d) \]

REFERENCES